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Interactions of a $j = 1$ Boson in the $2(2j+1)$ Component Theory*

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Abstract

The amplitudes for boson-boson and fermion-boson interactions are calculated in the second order of perturbation theory in the Lobachevsky space. An essential ingredient of the used model is the Weinberg's $2(2j+1)$ component formalism for describing a particle of spin j, recently developed substantially. The boson-boson amplitude is then compared with the two-fermion amplitude obtained long ago by Skachkov on the ground of the hamiltonian formulation of quantum field theory on the mass hyperboloid, $p_0^2 - \mathbf{p}^2 = M^2$, proposed by Kadyshevsky. The parametrization of the amplitudes by means of the momentum transfer in the Lobachevsky space leads to same spin structures in the expressions of T matrices for the fermion and the boson cases. However, certain differences are found. Possible physical applications are discussed.

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The scattering amplitude for the two-fermion interaction had been obtained in the Lobachevsky space in the second order of perturbation theory long ago [1a,Eq.(31)]:

$$T_V^{(2)}(\mathbf{k}(-)\mathbf{p},\mathbf{p}) = -g_v^2\frac{4m^2}{\mu^2+4\boldsymbol{æ}^2} - 4g_v^2\frac{(\boldsymbol{\sigma}_1\boldsymbol{æ})(\boldsymbol{\sigma}_2\boldsymbol{æ})-(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\boldsymbol{æ}^2}{\mu^2+4\boldsymbol{æ}^2} -$$
$$-\frac{8g_v^2p_0æ_0}{m^2}\frac{i\boldsymbol{\sigma}_1[\mathbf{p}\times\boldsymbol{æ}]+i\boldsymbol{\sigma}_2[\mathbf{p}\times\boldsymbol{æ}]}{\mu^2+4\boldsymbol{æ}^2} - \frac{8g_v^2}{m^2}\frac{p_0^2æ_0^2+2p_0æ_0(\mathbf{p}\cdot\boldsymbol{æ})-m^4}{\mu^2+4\boldsymbol{æ}^2} -$$
$$-\frac{8g_v^2}{m^2}\frac{(\boldsymbol{\sigma}_1\mathbf{p})(\boldsymbol{\sigma}_1\boldsymbol{æ})(\boldsymbol{\sigma}_2\mathbf{p})(\boldsymbol{\sigma}_2\boldsymbol{æ})}{\mu^2+4\boldsymbol{æ}^2}\quad , \tag{1}$$

g_v is the coupling constant. This treatment is based on the use of the formalism of separation of the Wigner rotations and parametrization of currents by means of the Pauli-Lyuban'sky vector, developed in the sixties [2]. The quantities

$$æ_0 = \sqrt{\frac{m(\Delta_0+m)}{2}}\quad , \quad \boldsymbol{æ} = \mathbf{n}_\Delta\sqrt{\frac{m(\Delta_0-m)}{2}}$$

are the components of the 4-vector of a momentum "half-transfer". This concept is closely connected with a notion of the half-velocity of a particle [3]. The 4-vector Δ_μ:

$$\boldsymbol{\Delta} = \boldsymbol{\Lambda}_{\mathbf{p}}^{-1}\mathbf{k} = \mathbf{k}(-)\mathbf{p} = \mathbf{k} - \frac{\mathbf{p}}{m}(k_0 - \frac{\mathbf{k}\cdot\mathbf{p}}{p_0+m})\quad , \tag{2a}$$
$$\Delta_0 = (\Lambda_p^{-1}k)_0 = (k_0p_0 - \mathbf{k}\cdot\mathbf{p})/m = \sqrt{m^2+\boldsymbol{\Delta}^2} \tag{2b}$$

could be regarded as the momentum transfer vector in the Lobachevsky space.[1, 2]

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[1]I keep a notation and a terminology of ref. [1]. *E.g.*, the vector current with taking into account the Pauli term is

$$j^\mu_{\sigma\sigma'}(\mathbf{p},\mathbf{k}) = \overline{u}_\sigma(\mathbf{p})\left\{g_v\gamma^\mu - f_v\frac{\sigma^{\mu\nu}}{2m}q_\nu\right\}u_{\sigma'}(\mathbf{k})\quad , \quad q=p-k \tag{3}$$

and

$$j^\mu_{\sigma\sigma'}(\mathbf{p},\mathbf{k}) = \sum_{\sigma_p=-1/2}^{1/2} j^\mu_{\sigma\sigma_p}(\mathbf{k}(-)\mathbf{p},\mathbf{p})\; D^{\frac{1}{2}}_{\sigma_p\sigma'}\left\{V^{-1}(\Lambda_p,k)\right\}\quad , \tag{4}$$

where $D^{\frac{1}{2}}_{\sigma_p\sigma'}\left\{V^{-1}(\Lambda_p,k)\right\} = \xi_{\sigma_p}D^{\frac{1}{2}}\left\{V^{-1}(\Lambda_p,k)\right\}\xi_{\sigma'}$; $D^{\frac{1}{2}}(A)\equiv D^{(\frac{1}{2},0)}(A)$ is the Wigner matrix of the irreducible representation of the $SU(2)$ group (or rotation group). The technique of construction of $D^J(A)$ could be found in ref. [4, p.51,70,English edition].

[2]In general, for each particle in interaction one should understand under 4-momenta p_i^μ and k_i^μ $(i=1,2)$ their covariant generalizations, $\overset{\circ}{p}{}_i^\mu$, $\overset{\circ}{k}{}_i^\mu$, *e.g.*, refs. [2,5,6]:

$$\overset{\circ}{\mathbf{k}} = (\boldsymbol{\Lambda}_{\mathcal{P}}^{-1}\mathbf{k}) = \mathbf{k} - \frac{\boldsymbol{\mathcal{P}}}{\sqrt{\mathcal{P}^2}}\left(k_0 - \frac{\boldsymbol{\mathcal{P}}\cdot\mathbf{k}}{\mathcal{P}_0+\sqrt{\mathcal{P}^2}}\right)\quad ,$$

This amplitude had been used for physical applications in the framework of the Kadyshevsky's version of the quasipotential approach [7,1].

From the other hand, in ref. [8,9] an attractive $2(2j+1)$ component formalism for describing particles of higher spins has been proposed. As opposed to the Proca 4-vector potentials, that transform according to the $(\frac{1}{2},\frac{1}{2})$ representation of the Lorentz group, the spinor $2(2j+1)$ component functions are constructed via the representation $(j,0)\oplus(0,j)$ in the Joos-Weinberg formalism. This way of description of higher spin particles is on an equal footing to the description of the Dirac spinor particle, whose wave function transforms according to the $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ representation. The $2(2j+1)$- component analogues of the Dirac functions in the momentum space are[3]

$$\mathcal{U}(\mathbf{p})=\sqrt{\frac{M}{2}}\begin{pmatrix} D^J\left(\alpha(\mathbf{p})\right)\xi_\sigma \\ D^J\left(\alpha^{-1\,\dagger}(\mathbf{p})\right)\xi_\sigma \end{pmatrix} \quad , \tag{5}$$

for the positive-energy states; and

$$\mathcal{V}(\mathbf{p})=\sqrt{\frac{M}{2}}\begin{pmatrix} D^J\left(\alpha(\mathbf{p})\Theta_{[1/2]}\right)\xi^*_\sigma \\ D^J\left(\alpha^{-1\,\dagger}(\mathbf{p})\Theta_{[1/2]}\right)(-1)^{2J}\xi^*_\sigma \end{pmatrix} \quad , \tag{6}$$

for the negative-energy states, ref. [4, p.107], with the following notations being used:

$$\alpha(\mathbf{p})=\frac{p_0+M+(\boldsymbol{\sigma}\cdot\mathbf{p})}{\sqrt{2M(p_0+M)}},\quad \Theta_{[1/2]}=-i\sigma_2 \quad . \tag{7}$$

For instance, in the case of spin $j=1$, one has

$$D^1\left(\alpha(\mathbf{p})\right) = 1+\frac{(\mathbf{J}\cdot\mathbf{p})}{M}+\frac{(\mathbf{J}\cdot\mathbf{p})^2}{M(p_0+M)} \quad , \tag{8a}$$

$$D^1\left(\alpha^{-1\,\dagger}(\mathbf{p})\right) = 1-\frac{(\mathbf{J}\cdot\mathbf{p})}{M}+\frac{(\mathbf{J}\cdot\mathbf{p})^2}{M(p_0+M)} \quad , \tag{8b}$$

$$D^1\left(\alpha(\mathbf{p})\Theta_{[1/2]}\right) = \left[1+\frac{(\mathbf{J}\cdot\mathbf{p})}{M}+\frac{(\mathbf{J}\cdot\mathbf{p})^2}{M(p_0+M)}\right]\Theta_{[1]} \quad , \tag{8c}$$

$$D^1\left(\alpha^{-1\,\dagger}(\mathbf{p})\Theta_{[1/2]}\right) = \left[1-\frac{(\mathbf{J}\cdot\mathbf{p})}{M}+\frac{(\mathbf{J}\cdot\mathbf{p})^2}{M(p_0+M)}\right]\Theta_{[1]} \quad , \tag{8d}$$

$$\overset{\circ}{k}_0=(\Lambda^{-1}_{\mathcal{P}}k)_0=\sqrt{m^2+\overset{\circ}{\mathbf{k}}{}^2},$$

with $\mathcal{P}=p_1+p_2$, $\Lambda^{-1}_{\mathcal{P}}\mathcal{P}=(\mathcal{M},\mathbf{0})$. However, we omit the circles above the momenta in the following, because in the case under consideration we do not miss physical information if use the corresponding quantities in c.m.s., $\mathbf{p}_1=-\mathbf{p}_2=\mathbf{p}$ and $\mathbf{k}_1=-\mathbf{k}_2=\mathbf{k}$.

[3] These functions obey the orthonormalization equations, $\mathcal{U}^\dagger(\mathbf{p})\gamma_{00}\mathcal{U}(\mathbf{p})=M$, M is the mass of the Joos-Weinberg particle. The similar normalization condition exists for $\mathcal{V}(\mathbf{p})$, the functions of "negative-energy states".

($\Theta_{[1/2]}, \Theta_{[1]}$ are the Wigner operators for spin 1/2 and 1, respectively). In spite of some antiquity of this formalism, in our opinion, it does not deserve to be retired. From the phenomenological viewpoint this approach provides the necessary framework of constructing a QCD-based effective field theory of higher-spin hadronic resonances and could yield new insights into the quark structure of these excited hadrons. Recently, much attention has been paid to this formalism [10,11] (see also the papers [12,13] regarding similar problems). Unfortunately, all the authors of the old classic works devoted to this formalism have missed the possibility of another definition of negative-energy bispinors $\mathcal{V}(\mathbf{p}) = S^c_{[1]}\mathcal{U}(\mathbf{p}) \equiv \mathcal{C}_{[1]}\mathcal{K}\mathcal{U}(\mathbf{p}) \sim \gamma_5\mathcal{U}(\mathbf{p})$, like the Dirac $j = 1/2$ case.[4] $S^c_{[1]}$ is the charge conjugation operator for $j = 1$, ref. [10c]; $\mathcal{K}$ is the operation of complex conjugation. This definition, based on the use of another form of the Ryder-Burgard relation [10c,d], leads to different physical content: in the latter case a boson and its antiboson have opposite relative intrinsic parities (like Dirac spinor particles). This is an example of another class of Poincarè invariant theories (the Bargmann-Wightman-Wigner-type quantum field theories [16]). This remarkable fact, which has been proven in refs. [10c,d], hints that the problem of the adequate choice of the field operator has profound physical significance.

In refs. [9,17,4,18,19] the Feynman diagram technique was discussed in the above-mentioned six-component formalism for particles of spin $j = 1$. The following Lagrangian:[5,6]

$$\begin{aligned}\mathcal{L} = \overline{\Psi}(x)\Gamma_{\mu\nu}\overleftarrow{\nabla}_\mu\overrightarrow{\nabla}_\nu\Psi(x) - M^2\overline{\Psi}(x)\Psi(x) - \frac{1}{4}F_{\mu\nu}F_{\mu\nu}+\\ + \frac{e\lambda}{12}F_{\mu\nu}\overline{\Psi}(x)\gamma_{5,\mu\nu}\Psi(x) + \frac{e\kappa}{12M^2}\partial_\alpha F_{\mu\nu}\overline{\Psi}(x)\gamma_{6,\mu\nu,\alpha\beta}\nabla_\beta\Psi(x)\end{aligned} \tag{9}$$

has been used there. In the above formula we have $\overleftrightarrow{\nabla}_\mu = -i\partial_\mu \mp eA_\mu$; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor; A_μ is the 4-vector of electromagnetic field; $\overline{\Psi}, \Psi$ are the six-component wave functions (WF) of a massive $j = 1$ Joos-Weinberg particle. The following expression has been obtained for the interaction vertex of the particle with a photon described by the vector potential, ref. [17,18]:

$$-e\Gamma_{\alpha\beta}(p+k)_\beta - \frac{ie\lambda}{6}\gamma_{5,\alpha\beta}q_\beta + \frac{e\kappa}{6M^2}\gamma_{6,\alpha\beta,\mu\nu}q_\beta q_\mu(p+k)_\nu \quad , \tag{10}$$

where $\Gamma_{\alpha\beta} = \gamma_{\alpha\beta}+\delta_{\alpha\beta}$; $\gamma_{\alpha\beta}$; $\gamma_{5,\alpha\beta}$; $\gamma_{6,\alpha\beta,\mu\nu}$ are the $6\otimes 6$-matrices which have been described in ref. [20,9]:

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[4]We don't treat here the new Majorana-like constructs in the $(j,0)\oplus(0,j)$ representation space [14] referring the reader to our recent works [15].

[5]In the following I prefer to use the Euclidean metric because this metric got application in a lot of papers on the $2(2j+1)$ formalism.

[6]The expression for the Lagrangian has been generalized in refs. [13]. In this paper we are still going to use the previous one in order to emphasize features of the formalism relevant to the purposes of the work.

$$\gamma_{ij} = \begin{pmatrix} 0 & \delta_{ij}\mathbb{1} - J_iJ_j - J_jJ_i \\ \delta_{ij}\mathbb{1} - J_iJ_j - J_jJ_i & 0 \end{pmatrix} \quad , \tag{11a}$$

$$\gamma_{i4} = \gamma_{4i} = \begin{pmatrix} 0 & iJ_i \\ -iJ_i & 0 \end{pmatrix} \quad , \quad \gamma_{44} = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \quad , \tag{11b}$$

and

$$\gamma_{5,\alpha\beta} = i[\gamma_{\alpha\mu}, \gamma_{\beta\mu}]_- \quad , \tag{12a}$$

$$\gamma_{6,\alpha\beta,\mu\nu} = [\gamma_{\alpha\mu}, \gamma_{\beta\nu}]_+ + 2\delta_{\alpha\mu}\delta_{\beta\nu} - [\gamma_{\beta\mu}, \gamma_{\alpha\nu}]_+ - 2\delta_{\beta\mu}\delta_{\alpha\nu} \quad . \tag{12b}$$

J_i are the spin matrices for a $j = 1$ particle, e is the electron charge, λ and κ are the quantities that correspond to the magnetic dipole moment and the electric quadrupole moment, respectively.

In order to obtain the 4-vector current for the interaction of a Joos-Weinberg boson with the external field one can use the known formulas of refs. [1,2], which are valid for any spin:

$$\mathcal{U}^\sigma(\mathbf{p}) = \boldsymbol{S}_{\mathbf{p}}\mathcal{U}^\sigma(\mathbf{0}) \quad , \quad \boldsymbol{S}_{\mathbf{p}}^{-1}\boldsymbol{S}_{\mathbf{k}} = \boldsymbol{S}_{\mathbf{k}(-)\mathbf{p}} \cdot I \otimes D^1\left\{V^{-1}(\Lambda_p, k)\right\} \quad , \tag{13}$$

$$W_\mu(\mathbf{p}) \cdot D\left\{V^{-1}(\Lambda_p, k)\right\} = D\left\{V^{-1}(\Lambda_p, k)\right\} \cdot \left[W_\mu(\mathbf{k}) + \frac{p_\mu + k_\mu}{M(\Delta_0 + M)} p_\nu W_\nu(\mathbf{k})\right] , \tag{14}$$

$$k_\mu W_\mu(\mathbf{p}) \cdot D\left\{V^{-1}(\Lambda_p, k)\right\} = -D\left\{V^{-1}(\Lambda_p, k)\right\} \cdot p_\mu W_\mu(\mathbf{k}) \quad . \tag{15}$$

W_μ is the Pauli-Lyuban'sky 4-vector of relativistic spin. The matrix $D^J\left\{V^{-1}(\Lambda_p, k)\right\}$ is written for spin $j = 1$ as follows:

$$\begin{aligned} D^{(j=1)}\left\{V^{-1}(\Lambda_p, k)\right\} &= \frac{1}{2M(p_0 + M)(k_0 + M)(\Delta_0 + M)}\Big\{[\mathbf{p}\times\mathbf{k}]^2 + \\ &+ [(p_0 + M)(k_0 + M) - \mathbf{k}\cdot\mathbf{p}]^2 + 2i\,[(p_0 + M)(k_0 + M) - \mathbf{k}\cdot\mathbf{p}]\,\{\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}]\} - \\ &- 2\{\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}]\}^2\Big\} \quad . \end{aligned} \tag{16}$$

However, the formulas obtained in ref. [19]:[7]

$$\boldsymbol{S}_{\mathbf{p}}^{-1}\gamma_{\mu\nu}\boldsymbol{S}_{\mathbf{p}} = \gamma_{44}\left\{\delta_{\mu\nu} - \frac{1}{M^2}\chi_{[\mu\nu]}(\mathbf{p}) \otimes \gamma_5 - \frac{2}{M^2}\Sigma_{[\mu\nu]}(\mathbf{p})\right\} \quad , \tag{17a}$$

$$\boldsymbol{S}_{\mathbf{p}}^{-1}\gamma_{5,\mu\nu}\boldsymbol{S}_{\mathbf{p}} = 6i\left\{-\frac{1}{M^2}\chi_{(\mu\nu)}(\mathbf{p}) \otimes \gamma_5 + \frac{2}{M^2}\Sigma_{(\mu\nu)}(\mathbf{p})\right\} \quad , \tag{17b}$$

where

[7] Attention is drawn to the definition of γ_5 matrix which differs by a sign from the definition used in refs. [17c,19].

$$\chi_{[\mu\nu]}(\mathbf{p}) = p_\mu W_\nu(\mathbf{p}) + p_\nu W_\mu(\mathbf{p}) \quad , \tag{18a}$$

$$\chi_{(\mu\nu)}(\mathbf{p}) = p_\mu W_\nu(\mathbf{p}) - p_\nu W_\mu(\mathbf{p}) \quad , \tag{18b}$$

$$\Sigma_{[\mu\nu]}(\mathbf{p}) = \frac{1}{2}\left\{W_\mu(\mathbf{p})W_\nu(\mathbf{p}) + W_\nu(\mathbf{p})W_\mu(\mathbf{p})\right\} \quad , \tag{18c}$$

$$\Sigma_{(\mu\nu)}(\mathbf{p}) = \frac{1}{2}\left\{W_\mu(\mathbf{p})W_\nu(\mathbf{p}) - W_\nu(\mathbf{p})W_\mu(\mathbf{p})\right\} \quad , \tag{18d}$$

lead to the 4- current of a $j=1$ Joos-Weinberg particle more directly:[8]

$$j_\mu^{\sigma_p\nu_p}(\mathbf{p},\mathbf{k}) = j_{\mu\,(S)}^{\sigma_p\nu_p}(\mathbf{p},\mathbf{k}) + j_{\mu\,(V)}^{\sigma_p\nu_p}(\mathbf{p},\mathbf{k}) + j_{\mu\,(T)}^{\sigma_p\nu_p}(\mathbf{p},\mathbf{k}) \quad , \tag{22a}$$

$$j_{\mu\,(S)}^{\sigma_p\nu_p}(\mathbf{p},\mathbf{k}) = -g_S\xi_{\sigma_p}^\dagger\left\{(p+k)_\mu\left(1+\frac{(\mathbf{J}\cdot\mathbf{\Delta})^2}{M(\Delta_0+M)}\right)\right\}\xi_{\nu_p} \quad , \tag{22b}$$

$$j_{\mu\,(V)}^{\sigma_p\nu_p}(\mathbf{p},\mathbf{k}) = -g_V\xi_{\sigma_p}^\dagger\left\{(p+k)_\mu + \frac{1}{M}W_\mu(\mathbf{p})(\mathbf{J}\cdot\mathbf{\Delta}) - \frac{1}{M}(\mathbf{J}\cdot\mathbf{\Delta})W_\mu(\mathbf{p})\right\}\xi_{\nu_p} \quad , \tag{22c}$$

$$\begin{aligned} j_{\mu\,(T)}^{\sigma_p\nu_p}(\mathbf{p},\mathbf{k}) = -g_T\xi_{\sigma_p}^\dagger\Big\{&-(p+k)_\mu\frac{(\mathbf{J}\cdot\mathbf{\Delta})^2}{M(\Delta_0+M)}+ \\ &+\frac{1}{M}W_\mu(\mathbf{p})(\mathbf{J}\cdot\mathbf{\Delta}) - \frac{1}{M}(\mathbf{J}\cdot\mathbf{\Delta})W_\mu(\mathbf{p})\Big\}\xi_{\nu_p} \quad . \end{aligned} \tag{22d}$$

Let us note an interesting feature [10b,13c]. The 6-spinors $\mathcal{U}(\mathbf{p})$ and $\mathcal{V}(\mathbf{p})$ defined by Eqs. (5,6) do not form a complete set:

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[8] *Cf.* with a $j=1/2$ case:

$$\boldsymbol{S}_p^{-1}\gamma_\mu\boldsymbol{S}_p = \frac{1}{m}\gamma_0\left\{\mathbb{1}\otimes p_\mu + 2\gamma_5\otimes W_\mu(\mathbf{p})\right\} \quad , \tag{19a}$$

$$\boldsymbol{S}_p^{-1}\sigma_{\mu\nu}\boldsymbol{S}_p = -\frac{4}{m^2}\mathbb{1}\otimes\Sigma_{(\mu\nu)}(\mathbf{p}) + \frac{2}{m^2}\gamma_5\otimes\chi_{(\mu\nu)}(\mathbf{p}) \quad , \tag{19b}$$

and, then,

$$j_\mu^{\sigma_p\nu_p}(\mathbf{k}(-)\mathbf{p},\mathbf{p}) = \frac{1}{m}\xi_{\sigma_p}^\dagger\left\{2g_v æ_0 p_\mu + f_v æ_0 q_\mu + 4g_{\mathcal{M}}W_\mu(\mathbf{p})(\boldsymbol{\sigma}\cdot\mathbf{æ})\right\}\xi_{\nu_p} \quad , \quad (g_{\mathcal{M}} = g_v + f_v) \quad . \tag{20}$$

The indices p indicate that the Wigner rotations have been separated out and, thus, all spin indices have been "resetted" on the momentum $\mathbf{p}$. One can re-write [1b] the electromagnetic current (20):

$$j_\mu^{\sigma_p\nu_p}(\mathbf{k},\mathbf{p}) = -\frac{e\,m}{æ_0}\xi_{\sigma_p}^\dagger\left\{g_{\mathcal{E}}(q^2)\,(p+k)^\mu + g_{\mathcal{M}}(q^2)\,\left[\frac{1}{m}W_\mu(\mathbf{p})(\boldsymbol{\sigma}\cdot\mathbf{\Delta}) - \frac{1}{m}(\boldsymbol{\sigma}\cdot\mathbf{\Delta})W_\mu(\mathbf{p})\right]\right\}\xi_{\nu_p} \quad . \tag{21}$$

$g_{\mathcal{E}}$ and $g_{\mathcal{M}}$ are the analogues of the Sachs electric and magnetic form factors. Thus, if we regard $g_{S,T,V}$ as effective coupling constants depending on the momentum transfer one can ensure ourselves that the form of the currents for a spinor particle and for a $j=1$ boson is the same (with Wigner rotations separated out).

$$\frac{1}{M}\left\{\mathcal{U}(\mathbf{p})\overline{\mathcal{U}}(\mathbf{p})+\mathcal{V}(\mathbf{p})\overline{\mathcal{V}}(\mathbf{p})\right\} = \begin{pmatrix} \mathbb{1} & \boldsymbol{S}_{\mathbf{p}}\otimes\boldsymbol{S}_{\mathbf{p}} \\ \boldsymbol{S}_{\mathbf{p}}^{-1}\otimes\boldsymbol{S}_{\mathbf{p}}^{-1} & \mathbb{1} \end{pmatrix} \quad . \tag{23}$$

But, if regard $\mathcal{V}_2(\mathbf{p})=\gamma_5\mathcal{V}(\mathbf{p})$ one can obtain the complete set. Fortunately,

$$\overline{\mathcal{V}}_2(\mathbf{0})\mathcal{U}_1(\mathbf{0})=0 \quad , \tag{24}$$

what permits us to keep the parametrization (4). As a matter of fact, in Eqs. (22b-22d) we have used the second definition of negative energy spinors, ref. [10c,d].

Next, let me now represent the Feynman matrix element corresponding to the diagram of two-boson interaction, mediated by the particle described by the vector potential, in the form [1,18] (read the remark in the footnote # 2):

$$\begin{aligned}
&< p_1, p_2;\, \sigma_1,\sigma_2|\hat{T}^{(2)}|k_1,k_2;\,\nu_1,\nu_2 >= \\
&= \sum_{\sigma_{ip},\nu_{ip},\nu_{ik}=-1}^{1} D^{\dagger\,(j=1)}_{\sigma_1\sigma_{1p}}\left\{V^{-1}(\Lambda_{\mathcal{P}},p_1)\right\} D^{\dagger\,(j=1)}_{\sigma_2\sigma_{2p}}\left\{V^{-1}(\Lambda_{\mathcal{P}},p_2)\right\}\times \\
&\times T^{\nu_{1p}\nu_{2p}}_{\sigma_{1p}\sigma_{2p}}(\mathbf{k}(-)\mathbf{p},\mathbf{p}) D^{(j=1)}_{\nu_{1p}\nu_{1k}}\left\{V^{-1}(\Lambda_{p_1},k_1)\right\} D^{(j=1)}_{\nu_{1k}\nu_1}\left\{V^{-1}(\Lambda_{\mathcal{P}},k_1)\right\}\times \\
&\times D^{(j=1)}_{\nu_{2p}\nu_{2k}}\left\{V^{-1}(\Lambda_{p_2},k_2)\right\} D^{(j=1)}_{\nu_{2k}\nu_2}\left\{V^{-1}(\Lambda_{\mathcal{P}},k_2)\right\} \quad ,
\end{aligned} \tag{25}$$

where

$$T^{\nu_{1p}\nu_{2p}}_{\sigma_{1p}\sigma_{2p}}(\mathbf{k}(-)\mathbf{p},\mathbf{p}) = \xi^\dagger_{\sigma_{1p}}\xi^\dagger_{\sigma_{2p}}\,T^{(2)}(\mathbf{k}(-)\mathbf{p},\,\mathbf{p})\,\xi_{\nu_{1p}}\xi_{\nu_{2p}} \quad , \tag{26}$$

$\xi^\dagger$, ξ are the analogues of Pauli spinors. The calculation of the amplitude (26) yields ($p_0 = -ip_4$, $\Delta_0=-i\Delta_4$):

$$\begin{aligned}
\hat{T}^{(2)}(\mathbf{k}(-)\mathbf{p},\mathbf{p}) \;=\; & g^2\left\{\frac{\left[p_0(\Delta_0+M)+(\mathbf{p}\cdot\boldsymbol{\Delta})\right]^2-M^3(\Delta_0+M)}{M^3(\Delta_0-M)}+\right. \\
&+\frac{i(\mathbf{J}_1+\mathbf{J}_2)\cdot[\mathbf{p}\times\boldsymbol{\Delta}]}{\Delta_0-M}\left[\frac{p_0(\Delta_0+M)+\mathbf{p}\cdot\boldsymbol{\Delta}}{M^3}\right]+\frac{(\mathbf{J}_1\cdot\boldsymbol{\Delta})(\mathbf{J}_2\cdot\boldsymbol{\Delta})-(\mathbf{J}_1\cdot\mathbf{J}_2)\boldsymbol{\Delta}^2}{2M(\Delta_0-M)}- \\
&\left.-\frac{1}{M^3}\frac{\mathbf{J}_1\cdot[\mathbf{p}\times\boldsymbol{\Delta}]\;\;\mathbf{J}_2\cdot[\mathbf{p}\times\boldsymbol{\Delta}]}{\Delta_0-M}\right\} \quad .
\end{aligned} \tag{27}$$

We have assumed $g_S=g_V=g_T$ above, what is motivated by group-theoretical reasons and by the analogy discussed in the footnote # 8. The expression (27) reveals the advantages of the $2(2j+1)$- formalism, since it looks like the amplitude for the interaction of two spinor particles with the substitutions

$$\frac{1}{2M(\Delta_0-M)}\Rightarrow\frac{1}{\boldsymbol{\Delta}^2}\quad\text{and}\quad\mathbf{J}\Rightarrow\boldsymbol{\sigma} \quad .$$

The calculations hint that many analytical results produced for a Dirac fermion could be applicable to describing a $2(2j+1)$ particle. Nevertheless, it is required adequate explanation of the obtained difference. An inquisitive reader could note: its origin lies at the kinematical level. Free-space (without interaction) Joos-Weinberg equations admit acausal tachyonic solutions [11]. "Interaction introduced in the massive Weinberg equations will couple to

both the causal and acausal solutions and thus cannot give physically reasonable results". However, let us not forget that we have used the Tucker-Hammer approach [17b], indeed, that does not possess tachyonic solutions.[9]

For the sake of completeness I also present the amplitudes for interaction of $j=1$ and $j=0$ particles, $j=1/2$ and $j=0$ particles, and $j=1/2$ and $j=1$ particles. Let us use the equation for the 4-current of spinor particle ($f_v=0$), defined by the formula (21); the equation (22b) with $\mathbf{J}=0$, for a scalar particle (*e.g.*, ref. [21]); the equation (22c) for the 4-current of a $j=1$ particle in the Joos-Weinberg formalism. Following to the technique of "resetting" polarization indices, we obtain in a first case:

$$\begin{aligned} \hat{T}^{(2)}_{0\,1}(\mathbf{k},\mathbf{p}) \;=\; &-\frac{g_0 g_1}{2m_1(\Delta_1^0-m_1)}\Big\{-2m_1^2(\Delta_1^0+m_1)+ \\ &+\Big[p_1^0(\Delta_1^0+m_1)+(\mathbf{p}\cdot\mathbf{\Delta}_1)\Big](p_1^0+p_2^0+k_1^0+k_2^0)+i\mathbf{J}\cdot[\mathbf{p}\times\mathbf{\Delta}_1](p_1^0+p_2^0+k_1^0+k_2^0)\Big\} \quad , \end{aligned} \tag{28}$$

what has a similar form to $\hat{T}^{(2)}_{0\frac{1}{2}}(\mathbf{k},\mathbf{p})$, which is below.

As a result of lengthy calculations one can write the boson-fermion amplitudes in the following form:

$$\begin{aligned} \hat{T}^{(2)}_{0\frac{1}{2}}(\mathbf{k},\mathbf{p}) \;=\; &-\frac{2g_0 g_{\frac{1}{2}}}{(2m_1)^{3/2}(\Delta_1^0-m_1)\sqrt{\Delta_1^0+m_1}}\Big\{-2m_1^2(\Delta_1^0+m_1)+ \\ &+\Big[p_1^0(\Delta_1^0+m_1)+(\mathbf{p}\cdot\mathbf{\Delta}_1)\Big](p_1^0+p_2^0+k_1^0+k_2^0)+i\boldsymbol{\sigma}\cdot[\mathbf{p}\times\mathbf{\Delta}_1](p_1^0+p_2^0+k_1^0+k_2^0)\Big\} \quad . \end{aligned} \tag{29}$$

and

$$\begin{aligned} \hat{T}^{(2)}_{1\frac{1}{2}}(\mathbf{p},\mathbf{k}) \;=\; &-\frac{2g_1 g_{\frac{1}{2}}}{(2m_1)^{3/2}(\Delta_1^0-m_1)\sqrt{\Delta_1^0+m_1}}\Big\{-2m_1^2(\Delta_1^0+m_1)+ \\ &+\Big[p_1^0(\Delta_1^0+m_1)+(\mathbf{p}\cdot\mathbf{\Delta}_1)\Big]\left(p_1^0+p_2^0+k_1^0+k_2^0\right)+i\boldsymbol{\sigma}_1\cdot[\mathbf{p}\times\mathbf{\Delta}_1](p_1^0+p_2^0+k_1^0+k_2^0)- \\ &-\frac{i}{m_2}\mathbf{J}_2\cdot[\mathbf{p}\times\mathbf{\Delta}_2]\left[(p_1^0+p_2^0)(\Delta_1^0+m_1)+\frac{(\mathbf{p}\cdot\mathbf{\Delta}_1)(p_1^0+p_2^0+m_1+m_2)^2}{2(p_1^0+m_1)(p_2^0+m_2)}\right]- \\ &-m_1\left[(\boldsymbol{\sigma}_1\cdot\mathbf{\Delta}_2)(\mathbf{J}_2\cdot\mathbf{\Delta}_1)-(\boldsymbol{\sigma}_1\cdot\mathbf{J}_2)(\mathbf{\Delta}_1\cdot\mathbf{\Delta}_2)+i\mathbf{J}_2\cdot[\mathbf{\Delta}_1\times\mathbf{\Delta}_2]\right]+ \\ &+\boldsymbol{\sigma}_1\cdot[\mathbf{p}\times\mathbf{\Delta}_1]\;\mathbf{J}_2\cdot[\mathbf{p}\times\mathbf{\Delta}_2]\frac{(p_1^0+p_2^0+m_1+m_2)^2}{2m_2(p_1^0+m_1)(p_2^0+m_2)}\Big\} \quad . \end{aligned} \tag{30}$$

Above we have used the notation:

$$\mathbf{\Delta}_1=\mathbf{k}-\frac{\mathbf{p}}{m_1}\left(k_1^0-\frac{\mathbf{k}\cdot\mathbf{p}}{p_1^0+m_1}\right) \quad , \quad \Delta_1^0=\sqrt{\mathbf{\Delta}_1^2+m_1^2} \quad , \tag{31a}$$

$$\mathbf{\Delta}_2=\mathbf{k}-\frac{\mathbf{p}}{m_2}\left(k_2^0-\frac{\mathbf{k}\cdot\mathbf{p}}{p_2^0+m_2}\right) \quad , \quad \Delta_2^0=\sqrt{\mathbf{\Delta}_2^2+m_2^2} \quad , \tag{31b}$$

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[9] I am not going to deal further with this subject in the present paper. The description of dynamics based on new kinematical ground [10] will be given in a detailed publication.

and $p_1^0 = \sqrt{\mathbf{p}^{\,2} + m_1^2}$, $k_1^0 = \sqrt{\mathbf{k}^{\,2} + m_1^2}$, $p_2^0 = \sqrt{\mathbf{p}^{\,2} + m_2^2}$, $k_1^0 = \sqrt{\mathbf{k}^{\,2} + m_2^2}$.

Discussion and possible physical applications:

The main result of this paper is the boson-boson amplitude calculated in the framework of the Joos-Weinberg theory. The separation of the Wigner rotations permits us to reveal certain similarities with a $j = 1/2$ case. Thus, this result provide a ground for the conclusion: if existence of the Joos-Weinberg bosons would be proven[10] many calculations produced earlier for fermion-fermion interactions mediated by the vector potential could be applicable to processes involving this matter structure. Moreover, the main result of the paper gives a certain hope at a possibility of the unified description of fermions and bosons. Now, I realize that all the above-mentioned is not surprising. The principal features of describing the particle world on the base of relativistic quantum field theory are not in some special representation of the group, $(1/2,0) \oplus (0,1/2)$ or $(1,0) \oplus (0,1)$, or $(1/2,1/2)$, but in the Lorentz group itself. Several old papers, *e.g.*, ref. [22], and recent paper [23] can support this conclusion. However, the difference between denominators of the amplitudes necessitates us to undertake further study of the $(1/2,0) \oplus (0,1/2)$ and $(1,0) \oplus (0,1)$ representations. These representations, of course, are contained in the general scheme of Joos and Weinberg.

After an appearance of the paper [11,13] (see also ref. [24]) we seem to be forced to use equations of this approach:[11]

$$\left[\gamma^{\mu_1\mu_2\ldots\mu_{2j}}\partial_{\mu_1}\partial_{\mu_2}\ldots\partial_{\mu_{2j}} + M^{2j}\right]\Psi(x) = 0 \tag{32}$$

for describing higher-spin particles. In the framework of the standard Proca/Rarita-Schwinger approach we deal with many contradictions in the particle interpretation of the field transforming on the corresponding representations of the Lorentz group (*e.g.*, acausal solutions even at the kinematical level; the absence of the well-defined massless limit; the non-consistency after introduction of interactions; the longitudity of antisymmetric tensor field after quantization, what contradicts with the classical limit and with the Weinberg theorem; longitudinal non-propagating solutions of the Maxwell's equations which could lead to speculations on a "massive" photon, "tired" light [24] and the lost of renormalizability of modern quantum field models *etc*).

Secondly, to the moment it was realized that a gluon can be described as a massive particle with dynamical mass appearing due to existence of the color charge and the self-interaction. This treatment permits one to eliminate some contradictions in the results of calculations of the proton form factor and the effective coupling constant $\alpha_S(q^2)$ on the basis of QCD (for recent discussion see ref. [25]). Therefore, the presented amplitude could serve

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[10]As already mentioned in this paper and in refs. [10c,d], probably, the crucial experiment for the Joos-Weinberg boson could be based on the determination of relative intrinsic parities of a boson and its antiboson.

[11]See ref. [10c,d] for a discussion on the possible additional term $\wp_{u,v}$ at the mass term for integer spins.

as a base for describing the gluonium, the bound state of two massive gluons. The fermion-boson amplitudes could be applied to describing the quark-diquark composite system.

For thirty years the quasipotential approach to quantum field theory [26,7] are regarded to be the most convenient and sufficiently general formalism for calculation of energy spectra of composite states. In the Bethe-Salpeter approach one has a non-physical parameter (relative time), difficulties with the normalization of the bound state wave function *etc.* All this necessitates us to introduce constraints on the wave function. As a matter of fact, they lead to the approaches which are equivalent to the quasipotential one. For recent discussion see ref. [27,28]. Therefore, one can use equations for the equal-time (quasipotential) wave function to achieve the goals discussed above. *E.g.*, for a composite system formed by fermion and boson of non-equal mass the equation was given (in the Kadyshevsky version of quasipotential approach) in [29]:

$$
\begin{aligned}
&2\mathring{p}_2^0\left(\mathcal{M}-\mathring{p}_1^0-\mathring{p}_2^0\right)\Phi_{\sigma_1\sigma_2}(\mathring{\mathbf{p}}) = \\
&= \frac{1}{(2\pi)^3}\sum_{\nu_1\nu_2}\int \frac{d^3\mathring{\mathbf{k}}}{2\mathring{k}_1^0}\, V^{\nu_1\nu_2}_{\sigma_1\sigma_2}(\mathring{\mathbf{p}},\mathring{\mathbf{k}})\,\Phi_{\nu_1\nu_2}(\mathring{\mathbf{k}}) \quad .
\end{aligned} \tag{33}
$$

Several works dealing with phenomenological description of hadrons in the $(j,0)\oplus(0,j)$ framework have been published [30,31] and submitted for publication [32].

Finally, not having any intentions to shadow theories based on the concept of vector potential, in our opinion, the principal question is not yet solved. It is not in formal advantages of one or another formalism for describing $j=1$ (or higher spin) particles, but in "the nature of Nature's mesons".

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